Robust Discretization of Flow in Fractured Porous Media

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Abstract
Flow in fractured porous media represent a challenge for discretization methods due to the disparate scales and complex geometry. Herein we propose a new discretization, based on the mixed finite element method and mortar methods. Our discretization handles complex, non-matching grids, and allows for fracture intersections and termination in a natural way. The discretization is applicable to both two and three spatial dimensions. Furthermore, we show how to obtain a finite volume variant in terms of cell-center pressures. Analysis shows the method to be first order convergent, which is sustained by the numerical examples.

The Method
Because of the complex structure of natural fracture networks, it remains a challenge to provide robust and flexible discretization methods. Here, we identify a few distinct features which are attractive from the perspective of applications. The method formulated in this work is specifically designed to meet these goals:

- Mass conservation. This is of particular significance when the flow field is coupled to transport (of heat, or composition), as transport schemes are typically very sensitive to non-conservative flow fields.
- Grid flexibility. This is important both in order to accommodate the structure of the fracture network, but also in order to honour other properties of the problem, such as material heterogeneities or anthropogenic features such as wells.
- Robustness. The method needs to be robust in the physically relevant limits. In the case of fractures, it is imperative to allow for arbitrary large aspect ratios, that is to say, thin fractures.

One of the key tools in the method is defining two mortar variables \( \lambda = (\lambda_1, \lambda_2) \) which represent the normal fluxes on both sides of the fracture. Then, an extension operator \( R \) is introduced so that the total flux can be represented by the sum of a function with zero normal flux \( (u_R) \) and this extension \( R \).

The governing equations consist of the mass conservation equation and Darcy’s law. Our formulation is novel in that it uses a mixed formulation in the mortar space, consisting of tangential fluxes and pressure.

The method is formulated hierarchically by distinguishing the different subdomains according to the dimensions. This allows for a unified treatment of the surroundings, fractures and intersections. The lower-dimensional features are assigned an aperture \( \epsilon \) and we will naturally also be interested in the intersection points.

The fractures can be modelled ranging from features with high permeability, stimulating channel flow, to features with low permeability which block the flow. The test case shows that the method is capable of handling fracture intersections, immersed fractures, and fractures crossing the domain. The solution contains no oscillations and clearly shows the expected stimulation and, respectively, blocking of flow in the two cases.

Convergence & Finite Volume Variant

Our analysis indicates that linear convergence is expected when choosing the discrete function spaces of lowest order. Numerical results confirm this, as shown by the table below.

Due to the mass-conserving properties of mixed formulations, the method allows for a finite volume variant. Due to the fewer degrees of freedom and the resulting positive definite system, the computational cost is significantly reduced, namely by a factor of 3. Nevertheless, the linear convergence shown by the lowest order, finite element method is maintained.

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References